Exercise 2.7.3

For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.

$$\dot{x} = \sin x$$

Solution

The potential function $V(x)$ satisfies

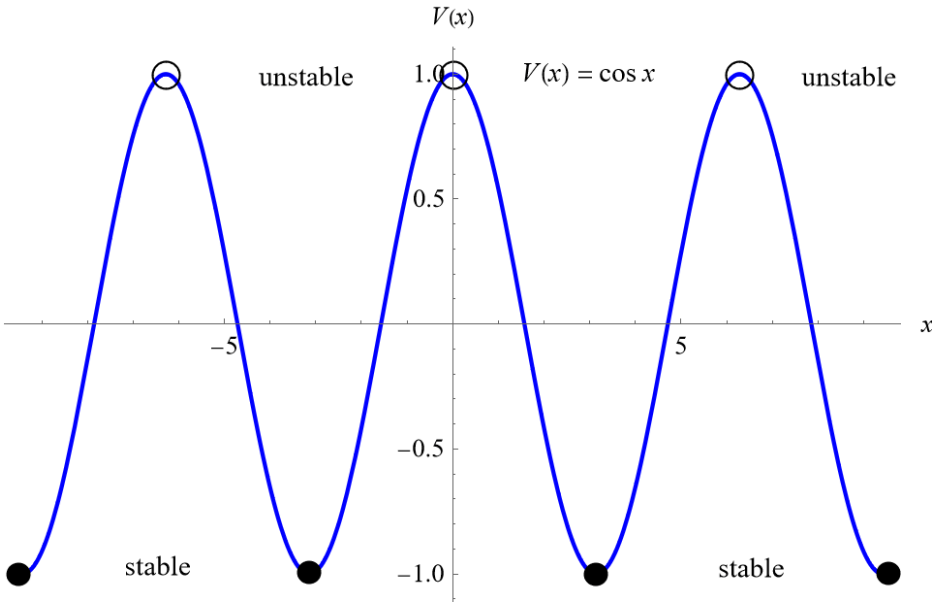
$$\dot{x} = \sin x = -\frac{dV}{dx}.$$

Multiply both sides by -1.

$$\frac{dV}{dx} = -\sin x$$

Integrate both sides with respect to x, setting the integration constant to zero.

$$V(x) = \cos x$$



The graph of $V(x)$ versus x is to be thought of as a two-dimensional rollercoaster. A particle on the curve at $x^* = 2n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$, is unstable because the slightest nudge in either direction will send it away from $x^* = 2n\pi$ indefinitely. A particle on the curve at $x^* = (2n+1)\pi$ that's nudged in either direction will return to $x^* = (2n+1)\pi$ because it's stable.